IOPscience iopscience.iop.org

Home Search Collections Journals About Contact us My IOPscience

Finite-size corrections of the $\mathbb{CP}^3$ giant magnons: the Lüscher terms

This article has been downloaded from IOPscience. Please scroll down to see the full text article.

JHEP07(2009)034

(http://iopscience.iop.org/1126-6708/2009/07/034)

The Table of Contents and more related content is available

Download details:
IP Address: 80.92.225.132
The article was downloaded on 03/04/2010 at 09:11

Please note that terms and conditions apply.

Published by IOP Publishing for SISSA

Received: *February 7, 2009*
Revised: *June 3, 2009*
Accepted: *June 13, 2009*
Published: *July 10, 2009*

Finite-size corrections of the $\mathbb{CP}^3$ giant magnons: the Lüscher terms

Diego Bombardelli and Davide Fioravanti

Sezione INFN di Bologna, Dipartimento di Fisica, Università di Bologna, Via Irnerio 46, Bologna, Italy

E-mail: bombardelli@bo.infn.it, fioravanti@bo.infn.it

Abstract: We compute classical and first quantum finite-size corrections to the recently found giant magnon solutions in two different subspaces of $\mathbb{CP}^3$. We use the Lüscher approach on the recently proposed exact S-matrix for $\mathcal{N} = 6$ superconformal Chern-Simons theory. We compare our results with the string and algebraic curve computations and find agreement, thus providing a non-trivial test for the new AdS_4/CFT_3 correspondence within an integrability framework.

Keywords: AdS-CFT Correspondence, Exact S-Matrix

ArXiv ePrint: 0810.0704

JHEP07(2009)034

© SISSA 2009

doi:10.1088/1126-6708/2009/07/034

Contents

JHEP07(2009)034

1 Introduction

In recent years, a great development in the AdS_5/CFT_4 correspondence [1] was put forward thanks to the discovery of integrable structures in both sides of this gauge/string duality (cf. for instance the seminal papers [2–4]).

Very recently, a new conjecture has been proposed regarding a correspondence between a large N M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$ and a three-dimensional $\mathrm{SU}(N) \times \mathrm{SU}(N)$ Chern-Simons matter theory whith $\mathcal{N} = 6$ superconformal symmetry [5].

Moreover, Minahan and Zarembo [6] have shown that this theory is integrable (see also [7]) at the second order in λ, that is a 't Hooft coupling defined as $\lambda = N/k$, which is made continuous when $N, k \to \infty$ and λ is kept fixed. On the string theory side, the integrability at the classical level has been shown in [8, 9].

Furthermore, giant magnon [18] solutions were found in the IIA string theory in $AdS_4 \times \mathbb{CP}^3$, dual to the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ sector of the gauge theory [19–22], and their finite-size effects were studied in [23–28].

On the other hand, an all-loop generalisation of the two-loop Bethe Ansatz proposed in [6] was conjectured by [10] and may be derived starting from a S-matrix proposed in [11].[1] Actually, the string theory computations [13–15] of the folded spinning string energy led to a result that was different from the Bethe Ansatz prediction of [10]. It was suggested in [16] that this disagreement was due to different regularisations used in the calculation of the one-loop correction to the string energy by the algebraic curve method. A resolution to this apparent contradiction between the worldsheet and the Bethe Ansatz calculations has

[1]See also [12] for the original derivation of the SYM Bethe Ansatz equations from the AdS_5/CFT_4 S-matrix

been recently proposed in [17], indicating that the Bethe Ansatz proposal may be correct at strong coupling. Since the dispute concerns one-loop results, we expect agreement in derivations at leading order at strong coupling, in which we are mainly interested in this paper.

In fact, the aim of this paper is to compute leading finite-size corrections in the simplest case of an elementary giant magnon (GM) through the generalised Lüscher method (see [29, 30, 32–35] regarding mainly AdS_5/CFT_4), based on scattering data (S-matrix).

These terms correct the infinite volume dispersion relation and should take into account possible wrapping effects (cf., for instance, [30, 31] for the more studied phenomenon of AdS_5/CFT_4 wrapping).

After the careful analysis in [20, 21], one can easily understand that there is a classical solution of GM kind in $\mathbb{CP}^3$, which lives in $\mathbb{R}_t \times S^2 \times S^2$ and whose infinite volume dispersion relation behaves at large λ in this way

$$\epsilon(p) \simeq 2\sqrt{2\lambda}\left|\sin\left(\frac{p}{2}\right)\right| \ . \tag{1.1}$$

This solution was interpreted in [21, 23] as composed by two magnons, each one in a S^2, with equal worldsheet momenta $p \equiv p_1 = p_2$ and the following infinite volume dispersion relation[2]

$$\epsilon_s(p) = \sqrt{\frac{1}{4} + 4h^2(\lambda)\sin^2\left(\frac{p}{2}\right)}\,, \tag{1.2}$$

where

$$h(\lambda) = \begin{cases} \lambda + O(\lambda^2) \ \text{for} \ \lambda \ll 1 \\ \\ \sqrt{\lambda/2} + O(\lambda^0) \ \text{for} \ \lambda \gg 1 \ . \end{cases} \tag{1.3}$$

Consistently, at large λ, (1.2) becomes one half of (1.1).

Therefore, in our calculations of the Lüscher terms, we will have to use the formulae for multiparticle states (see [36–39] for some applications in AdS_5/CFT_4).

On the other hand, [26] found, by algebraic curve methods, the first quantum correction to the energy of a GM that lives on $\mathbb{CP}^1 \approx S^2$, with the same dispersion relation of (1.2). In order to distinguish this solution, we will call it "small" GM.

We will show the calculations for the μ- and F-term of the GM in section 2 and 3, respectively. We find agreement with the string results for the μ-term and propose a new result for the first quantum finite-size correction, that is very similar, at the level of the final integral expression, to the algebraic curve result for the "big" GM, that lives in $\mathbb{RP}^2$, obtained in [26]. Clearly, this fact opens the way to the possible interpretation of the $\mathbb{RP}^2$ GM as given by a couple of SU(2) GMs. In section 4, we will present computations for the μ- and F-term of the "small" GM. In the latter case we will give a result which confirms the algebraic curve calculations [26], for the classical leading contribution, instead, we propose a new result that probably will require a deeper understanding. Finally, we also give some results for the next-to-leading contributions to the μ-terms in section 5. We conclude with some conclusions in section 6.

[2]We ought to thank G. Grignani for clarifying this point to us.

JHEP07(2009)034

2 The μ-term for the $\mathbb{R} \times S^2 \times S^2$ giant magnon

In this section we want to compute the leading finite-size correction to $\epsilon(p)$, $\delta\epsilon^\mu(p)$, as the Lüscher μ-term for a nonrelativistic theory characterised by a dispersion relation of kind (1.2). The generalisation of the Lüscher μ-term energy correction [29] for a single particle to a generic nonrelativistic theory, has been first derived by [32], and reads

$$\delta\epsilon_a^\mu = -i\left(1 - \frac{\epsilon'(p)}{\epsilon'(\tilde{q}^*)}\right) e^{-i\tilde{q}^* L} \underset{q^*=\tilde{q}^*}{Res} \sum_b (-1)^{F_b} S_{ba}^{ba}(q^*, p)\,, \tag{2.1}$$

where $\tilde{q}^*$ corresponds to the bound state pole of the S-matrix, p is the momentum of the real particle, denoted by a.

Now, since the GM solution on $\mathbb{R}_t \times S^2 \times S^2$ was interpreted in [21, 23] as a couple of two magnons with equal momenta, then, in order to calculate the finite-size correction, we have to reconsider generalised Lüscher formulae for multiparticle states [38] :

$$\delta\epsilon_A^\mu = -i\sum_{l=1}^{M}\sum_b (-1)^{F_b}\left(1 - \frac{\epsilon'_{a_l}(p_l)}{\epsilon'_b(\tilde{q}_l^*)}\right) e^{-i\tilde{q}_l^* L} \underset{q^*=\tilde{q}_l^*}{Res} S_{ba_l}^{ba_l}(q^*, p_l) \prod_{k\neq l}^{M} S_{ba_k}^{ba_k}(\tilde{q}_l^*, p_k)\,, \tag{2.2}$$

where $A \equiv \{a_1(p_1), \ldots, a_M(p_M)\}$ denotes a string made of M GMs.

We have to apply this formula to the case of two real particles - one of type A and the other of type B - interacting with another couple of virtual particles - of type A and B - moving around the cylinder. The S-matrices we will use to describe these interactions - between A-A, A-B and B-B particles - are those proposed in [11]:

$$\begin{aligned}
S^{AA}(p_1,p_2) &= S^{BB}(p_1,p_2) = S_0(p_1,p_2)\hat{S}(p_1,p_2) = \sigma(p_1,p_2)\,\frac{1-\frac{1}{x_1^+x_2^-}}{1-\frac{1}{x_1^-x_2^+}}\,\hat{S}(p_1,p_2)\\
S^{AB}(p_1,p_2) &= S^{BA}(p_1,p_2) = \tilde{S}_0(p_1,p_2)\hat{S}(p_1,p_2) = \sigma(p_1,p_2)\,\frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\,\hat{S}(p_1,p_2)\,,
\end{aligned} \tag{2.3}$$

where $\sigma(p_1, p_2)$ is the BES/BHL dressing factor [42–45], and $\hat{S}$ can expressed just as in the appendix A.5 of [49], through a set of functions $a_1, \ldots, a_{10}$ dependent on the variables $x_{1,2}^\pm$. We will choose the so-called string basis of the S-matrix, in order to obtain the correct string result, as since [32, 33]. Moreover, we take into account that only S^{AA}, S^{BB} have physical poles, corresponding to BPS bound states, determined by the same condition $x_q^- = x_p^+$.

In few words, the real A-particle scatters with the virtual A-particle and forms with it a bound state - corresponding to the physical pole of S^{AA} - while the B-particle scatters elastically with the virtual A-particle, because S^{AB} does not have physical poles, as we will see below. Then the real B-particle proceeds to form a bound state - corresponding to the physical pole of S^{BB} - with the virtual B-particle. Finally, we have to sum the contributions given by this diagram over all the possible residues of the S-matrix, taking into account that both real paricles belong to the SU(2) sector (i.e. $a_1 = a_2 = 1$). Moreover, in this case

JHEP07(2009)034

the real particles have equal momenta $p_1 = p_2 = p$, then $S^{AA}(q^*, p)$ and $S^{BB}(q^*, p)$ share the same pole. Hence, the above description shall suggest us this S-matrix contribution:

$$S^{AA}(q^*, p)S^{AB}(q^*, p) + S^{BA}(q^*, p)S^{BB}(q^*, p)\,, \tag{2.4}$$

on which we need to pick up the residues. Then we may propose the following expression for the μ-term of the $\mathrm{SU}(2)_A\times$ $\mathrm{SU}(2)_B$ giant magnon

$$\begin{aligned}\delta\epsilon^{\mu} \;=\; &-i\sum_b(-1)^{F_b}\left\{\left(1-\frac{\epsilon_1'(p)}{\epsilon_b'(\tilde q_1^*)}\right)e^{-i\tilde q_1^* L}\left[(S^{AB})^{b1}_{b1}(\tilde q_1^*,p)\operatorname*{Res}_{q^*=\tilde q_1^*}(S^{AA})^{b1}_{b1}(q^*,p)+\right.\right.\\ &\left.+\,(S^{BA})^{b1}_{b1}(\tilde q_1^*,p)\operatorname*{Res}_{q^*=\tilde q_1^*}(S^{BB})^{b1}_{b1}(q^*,p)\right]+\left(1-\frac{\epsilon_1'(p)}{\epsilon_b'(\tilde q_2^*)}\right)e^{-i\tilde q_2^* L}\times\\ &\left.\times\left[(S^{AB})^{b1}_{b1}(\tilde q_2^*,p)\operatorname*{Res}_{q^*=\tilde q_2^*}(S^{AA})^{b1}_{b1}(q^*,p)+(S^{BA})^{b1}_{b1}(\tilde q_2^*,p)\operatorname*{Res}_{q^*=\tilde q_2^*}(S^{BB})^{b1}_{b1}(q^*,p)\right]\right\}\,.\end{aligned}\tag{2.5}$$

Now, the evaluation of the various terms in (2.5) follows closely the derivation in [32], then we can omit the details of calculation. We have to take into account only the replacement of $g = \sqrt{\lambda_{SYM}}/4\pi$ with $h(\lambda)$ and the different structure of the S-matrix contribution:

$$\begin{aligned}&\sum_b(-1)^{F_b}(S^{AA})^{b1}_{b1}(q^*,p)(S^{AB})^{b1}_{b1}(q^*,p)+(S^{BB})^{b1}_{b1}(q^*,p)(S^{BA})^{b1}_{b1}(q^*,p)=\\ &\quad=2\,S_0(q^*,p)\tilde S_0(q^*,p)\left\{a_1^2(x_{q^*},x_p)+[a_1(x_{q^*},x_p)+a_2(x_{q^*},x_p)]^2-2\,a_6^2(x_{q^*},x_p)\right\}.\end{aligned}\tag{2.6}$$

that is of the form $\tilde S_0 S_0\sum_b(-1)^{F_b}(S^{b1}_{b1})^2$ rather than $\tilde S_0 S_0(\sum_b(-1)^{F_b}S^{b1}_{b1})^2$ of the SYM case. At the leading order in the strong coupling limit, in which only a_1 survives, this implies an extra factor 2 with respect to the SYM case. The other factor 2 in (2.6) is due simply to the equal contribution of the two terms in the l.h.s.

In conclusion, we obtain the result for the finite-size correction to the dispersion relation of a GM in $\mathrm{SU}(2)\times\mathrm{SU}(2)$, in perfect agreement with that given in equations (2) and (54) of [23]:

$$\delta\epsilon^{\mu} \simeq -\frac{8\sqrt{2\lambda}}{e^2}\sin^3\left(\frac{p}{2}\right)e^{-\frac{L}{\sqrt{2\lambda}\sin(p/2)}}\,. \tag{2.7}$$

Of course, it can be directly related to the analogue in SYM [32, 47, 48, 52]

$$\delta\epsilon^{\mu}_{SYM} \simeq -\frac{16}{e^2}\,g\sin^3\left(\frac{p}{2}\right)e^{-\frac{L}{\sqrt{2}g\sin(p/2)}}\,. \tag{2.8}$$

by substituting g with $h(\lambda) \simeq \sqrt{\lambda/2}$.

3 The F-term for the $\mathbb{R}\times S^2\times S^2$ giant magnon

Here we consider - as in the previous section - a GM with excitations on both S^2; then we have to take into account interactions between A and B particles inserting also S^{AB} in the final S-matrix. The generalised multi-particle formula for the Lüscher F-term is [38]

$$\delta E^F_A = -\sum_b(-1)^{F_b}P.V.\int_{-\infty}^{+\infty}\frac{dq}{2\pi}\left(1-\sum_{k=1}^{M}\alpha_k\,\frac{\epsilon'_{a_k}(p_k)}{\epsilon'_b(q^*)}\right)e^{-iq^*L}\left(\prod_{l=1}^{M}S^{ba_l}_{ba_l}(q^*,p_l)-1\right), \tag{3.1}$$

JHEP07(2009)034

where $\sum_{k=1}^{M} \alpha_k = 1$, and $P.V.$ indicates that we take the principal value of the integral.

We start from determining the kinematic part of the integral above. Firstly, the energy of the virtual particle with momentum q^* is parametrised by the variables $x^{\pm}_{q^*}$, that scale at strong coupling as [10]

$$x^{\pm}_{q^*} = x \pm \frac{i\,x^2}{2\,h(\lambda)(x^2-1)} + O(1/\lambda)\ . \tag{3.2}$$

Since also in this case the calculations - once we replace $g \to h(\lambda)$ - follow closely the derivation of the F-term in SYM [33], we give immediately the result for the kinematic part:

$$\partial_x \Omega(x) \equiv i\,\frac{dq}{dx}\left(1 - \frac{\epsilon'(p)}{\epsilon'(q^*(q))}\right) = \frac{1}{(x^2-1)^2}\left(-2x + (x^2+1)\,\frac{x_p^+ + x_p^-}{x_p^+\,x_p^- + 1}\right)\,, \tag{3.3}$$

where $\Omega(x)$ is usually defined in the algebraic curve approach as the function determining the characteristic frequencies of the energy fluctuations. In particular, definition (3.3) is satisfied by

$$\Omega(x) = \frac{1}{x^2-1}\left(1 - \frac{x_p^+ + x_p^-}{x_p^+\,x_p^- + 1}\,x\right)\,, \tag{3.4}$$

which coincides with the expression [26] valid for both "small" and "big" GM solutions.

The dressing part in (2.6) contributes with

$$S_0(q^*,p)\tilde{S}_0(q^*,p) = \frac{x^-_{q^*} - x^+_p}{x^+_{q^*} - x^-_p}\,\frac{1 - \frac{1}{x^+_{q^*}x^-_p}}{1 - \frac{1}{x^-_{q^*}x^+_p}}\,\sigma(x_{q^*},x_p)^2 \simeq e^{-\frac{i\sqrt{2}(\Delta-L)}{\sqrt{\lambda}}}\,, \tag{3.5}$$

where $\Delta = L + 2\sqrt{2\lambda}\sin(p/2)$, while at strong coupling the contribution of a_2 can be neglected at leading order, and then the "undressed" part in (2.6) results

$$2\,\left[a_1(x_{q^*},x_p)^2 - a_6(x_{q^*},x_p)^2\right] \simeq 2\left[\left(\frac{x - x_p^-}{x x_p^- - 1}\right)^2 - 1\right] \simeq \frac{4\,i\,e^{-ip/2}\sin(p/2)(x^2-1)}{\left(e^{ip/2}-x\right)^2}\,, \tag{3.6}$$

where the sign minus of the term $a_6(x_q,x_p)^2$ is due to the term $(-1)^{F_b}$, with $F_b = 0$ for bosonic and $F_b = 1$ for fermionic terms.

All together these terms give

$$\delta E^F \simeq 2P.V.\oint_{\mathbb{U}^+} \frac{dx}{\pi i}\partial_x\Omega(x)\,e^{-i\sqrt{2}\frac{\Delta}{\sqrt{\lambda}}\frac{x}{x^2-1}}\left[\left(\frac{x - x_p^-}{x\,x_p^- - 1}\right)^2 - 1\right]\,, \tag{3.7}$$

that, except for the parametrisation in terms of $p/2$ instead of $p/4$, coincides with the result of [26] for the "big" GM; then the result obtained with the algeraic curve and with the Lüscher techniques seems to be formally in agreement at all orders in $\sqrt{\lambda}/L$ (the situation here is exactly the same as in the AdS_5/CFT_4 correspondence [33]). Using the saddle

JHEP07(2009)034

point method, we can give an approximated evaluation of the above integral at leading order in $\sqrt{\lambda}/\Delta$:

$$\delta E^F \simeq -4\sqrt{\frac{\sqrt{\lambda/2}}{\pi\Delta}}\, e^{-\frac{\Delta}{\sqrt{2\lambda}}}\, \frac{\sin\left(\frac{p}{2}\right)}{1-\sin\left(\frac{p}{2}\right)}\,. \tag{3.8}$$

We immediately notice that our result (3.8), even after a replacement $p/2 \to p/4$, is different to the same quantity (5.12) in [26], although the integral expression for the F-term in the line before is the same in that paper.[3] We think that the reason of this discrepancy is the different integration curve in the complex plane adopted here. Here we have integrated only on the upper half of the unite circle, because the bijective map given by

$$q \simeq i\,\frac{x^2+1}{2\,(x^2-1)} \tag{3.9}$$

send the real axis to the upper half-circle, as explained in [33]. Therefore, we think that there is a mistake in [26] about the evaluation of this integral. We will explain this point in the appendix in a more detailed way.

Contrarily to the previous case, the dependence on p is very different if compared with the F-term of two GMs in SYM [33], which reads

$$\delta E^F_{SYM} \simeq -16\sqrt{\frac{g}{\pi\Delta_2^{SYM}}}\, e^{-\frac{\Delta_2^{SYM}}{2g}}\left(\frac{\sin^2\left(\frac{p}{2}\right)}{\left(\sin^2\left(\frac{p}{2}\right)-1\right)^2}\right), \tag{3.10}$$

where $\Delta_2^{SYM} = L + 8\,g\sin(p/2)$.

4 The μ- and the F-term of the $\mathbb{CP}^1$ giant magnon

Let us consider a single giant magnon which belongs to the $\mathrm{SU}(2)_A$ sector, for instance. If we take the formula (2.1) for the one-particle case and consider the S-matrix contribution[4]

$$S^{AA}(p_1,p_2) + S^{AB}(p_1,p_2) = \left(\frac{1-\frac{1}{x_1^+x_2^-}}{1-\frac{1}{x_1^-x_2^+}} + \frac{x_1^- - x_2^+}{x_1^+ - x_2^-}\right)\sigma(p_1,p_2)\,\hat{S}(p_1,p_2)\,, \tag{4.1}$$

then we have all the ingredients to compute the μ-term of this "small" GM.

Indeed, we have only to repeat the calculations of the section 2, without considering contributions by S^{AB}, but only by S^{AA}, since the latter only has a pole corresponding to a boundstate of two A-particles, that is a necessary condition to have a non-vanishing residue of the S-matrix, which determines the μ-term:

$$\sum_b (-1)^{F_b} \underset{q^*=\tilde{q}^*}{Res}\left(S^{AA}\right)_{b1}^{b1}(q_*,p) \simeq \frac{2}{x_q'^-}\,\frac{1-\frac{1}{x_q^+x_p^-}}{1-\frac{1}{x_q^-x_p^+}}\,\left(x_q^- - x_p^+\right)\, a_1\left(x_q,x_p\right)\,\sigma\left(x_q,x_p\right)\,. \tag{4.2}$$

---

[3] Except the dependence in p, once we replace $E_{Ref.[26]}(Q=1)$ by our Δ, the two results match at strong coupling.

[4] Actually the contribution of S^{AB} into the self-energy processes of a single A-particle, in a system of A-B particles, is considered also in QCD applications of the Lüscher terms: see, for instance, [53].

JHEP07(2009)034

We obtain the μ-term for the "small" GM at strong coupling

$$\delta\epsilon_s^{\mu} \simeq \frac{2\,i}{e}\sin\left(\frac{p}{2}\right)e^{-\frac{L}{\sqrt{2\lambda}\sin(p/2)}}\,, \tag{4.3}$$

that surprisingly is an imaginary quantity! We have to make a remark on this strange fact. At this point we could think that we should take the real part of this result, as stated in [37, 38], because of the replacement $\cos(q^* L) \to e^{-iq^* L}$ made in the derivation of μ- and F-term in [29, 32]. Alternatively, one could instead try to find a formulation - as in [35] form algebraic curve method - that could guarantee the reality of the whole - not expanded in $(1/\sqrt{\lambda})$ - expression (2.1). We hope, but at this moment we cannot demonstrate, that this is the case, and we reserve this problem to future investigations.

Now, we want to compute the F-term leading contibution for this "small" GM solution. Thus, in this case we can take the whole S-matrix contribution (4.1), so that we obtain

$$\sum_b(-1)^{F_b}\left[\left(S^{AA}+S^{AB}\right)\right]_{b1}^{b1}(q^*,p) = (2\,a_1+a_2-2\,a_6)\,\sigma(q^*,p)\left(\frac{1-\frac{1}{x_{q^*}^+ x_p^-}}{1-\frac{1}{x_{q^*}^- x_p^+}}+\frac{x_{q^*}^- - x_p^+}{x_{q^*}^+ - x_p^-}\right). \tag{4.4}$$

Taking the expressions in the appendix A of [33] for the strong coupling limit of the elements a_1, a_2, a_6 and for $\sigma(x_{q^*}, x_p)$, and using the expansion (3.2) for x_{q^*} and $x_p^{\pm} \simeq e^{\pm ip/2}$, the the previous equation becomes

$$\sum_b(-1)^{F_b}\left[\left(S^{AA}\right)_{b1}^{b1}+\left(S^{AB}\right)_{b1}^{b1}\right](q^*,p) \simeq 4\,e^{-i\sqrt{2}\frac{(\Delta_s-L)}{\sqrt{\lambda}}\frac{x}{x^2-1}}\left(\frac{x-x_p^-}{x_p^-\,x-1}-1\right)\,, \tag{4.5}$$

where $\Delta_s = L+\sqrt{2\lambda}\sin(p/2)$. In this way we obtain the following expression

$$\delta E_s^F = 2P.V.\oint_{\mathbb{U}^+}\frac{dx}{\pi i}\,\partial_x\Omega(x)\,e^{-i\sqrt{2}\frac{\Delta_s}{\sqrt{\lambda}}\frac{x}{x^2-1}}\left(\frac{x-x_p^-}{x_p^-\,x-1}-1\right)\,, \tag{4.6}$$

that agrees with the expression for the one loop finite-size correction of the "small" GM in [26]. If we proceed now to evaluate this integral via the usual saddle-point method, we find

$$\delta E_s^F \simeq -2\sqrt{\frac{\sqrt{\lambda/2}}{\pi\Delta_s}}\,e^{-\frac{\Delta_s}{\sqrt{2\lambda}}}\left(\frac{\cos(p/2)}{1-\sin(p/2)}-1\right)\,. \tag{4.7}$$

Obviously, the result of [26] is different from ours in the same way as in the previous section: there is a discrepancy in evaluating the final integral for the F-term (more details in the appendix).

Comparing with the SYM result [33], we can see that our expression for the first quantum correction to the finite- size effect is very different, because of the different dependence on the momentum:

$$\delta E_{SYM}^F \simeq -8\sqrt{\frac{g}{\pi\Delta^{SYM}}}\,e^{-\frac{\Delta^{SYM}}{2g}}\left(\frac{\cos\left(\frac{p}{2}\right)-1}{\sin\left(\frac{p}{2}\right)-1}\right)\,, \tag{4.8}$$

JHEP07(2009)034

where $\Delta^{SYM} = L + 4\,g\sin(p/2)$. However, the prefactor and the exponential term, as one can easily notice comparing the two expressions (4.7) and (4.8), can be mapped, except to the specific form of the Δs, to our result substituting g with the first order term of $h(\lambda)$ at strong coupling.

5 Next-to-leading contribution of the μ-terms

While considering the next-to-leading term, predicted to be a constant $c = -\ln(2)/2\pi$ by [17], in the strong coupling expansion of the central function $h(\lambda)$

$$h(\lambda) = \sqrt{\lambda/2} + c + O\left(\frac{1}{\sqrt{\lambda}}\right) \quad \text{for } \lambda \gg 1\,, \tag{5.1}$$

we may proceed to the expansion of the Zhukovsky variables $x^{\pm}_{p,q}$ up to the order $1/\lambda^{3/2}$:

$$x^{\pm}_{p} = e^{\pm ip/2}\left(1 + \frac{1}{2\sqrt{2\lambda}\sin(p/2)} + \frac{1}{16\lambda\sin^2(p/2)} - \frac{c}{2\lambda\sin(p/2)} + O\left(\frac{1}{\lambda^{3/2}}\right)\right) \tag{5.2}$$

$$x^{+}_{q} = e^{ip/2}\left(1 + \frac{3}{2\sqrt{2\lambda}\sin(p/2)} + i\,\frac{17 - e^{ip} - 48\,c\sin^2(p/2)}{32\,\lambda\, e^{ip/2}\sin^3(p/2)} + O\left(\frac{1}{\lambda^{3/2}}\right)\right)\,, \tag{5.3}$$

and x^-_q determined by the boundstate condition $x^-_q = x^+_p$. The exponential term is now given by

$$e^{-i\tilde{q}^*_{1,2}L} \equiv e^{-i\tilde{q}^* L} = e^{-\frac{L}{\sqrt{2\lambda}\sin(p/2)}}\left[1 - \frac{L}{2\lambda}\left(i\,\frac{\cos(p/2)}{\sin^3(p/2)} - \frac{c}{\sin(p/2)}\right)\right] + O\left(\frac{1}{\lambda^{3/2}}\right)\,, \tag{5.4}$$

while the kinematical factor reads

$$1 - \frac{\epsilon'_{a_{1,2}}(p_{1,2})}{\epsilon'_b(\tilde{q}^*)} = \sin^2\left(\frac{p}{2}\right) - \frac{i\,\cos(p/2)}{\sqrt{2\lambda}} + O\left(\frac{1}{\lambda^{3/2}}\right)\,. \tag{5.5}$$

It remains to evaluate the S-matrix contribution, that is given in part by the following limit

$$\lim_{q^*\to\tilde{q}^*}\left(\frac{q^* - \tilde{q}^*}{x^-_q - x^+_p}\right) = \frac{1}{x^{-\prime}_q} = \frac{i\,e^{-i\frac{p}{2}}}{2\sin^2\left(\frac{p}{2}\right)} - \frac{3 + 2\,e^{-ip}}{4\sqrt{2\lambda}\sin^4(p/2)} + O\left(\frac{1}{\lambda}\right)\,, \tag{5.6}$$

that is due to taking the residue in the momentum of the boundstate, in part by the "undressed" S-matrix elements

$$\frac{1 - \frac{1}{x^+_q x_p}}{1 - \frac{1}{x^-_q x^+_p}}\,\frac{(x^-_q - x^+_p)^2}{x^+_q - x^-_p}\,[a_1^2 + (a_1 + a_2)^2 - 2\,a_6^2] = \frac{2\sqrt{2}\,e^{3i\frac{p}{2}}}{\sqrt{\lambda}\sin\left(\frac{p}{2}\right)} + \frac{i\,e^{ip} - 4\,c\sin^2(p/2)}{\lambda\sin^3(p/2)} + O\left(\frac{1}{\lambda^{3/2}}\right) \tag{5.7}$$

and in part by the dressing factor

$$\begin{aligned}\sigma^2(x_q, x_p) = &-\frac{4\,\lambda}{e^2}\,e^{-ip}\sin^4\left(\frac{p}{2}\right) - \frac{2\sqrt{2\lambda}\,e^{-ip}\sin^2(p/2)}{\pi\,e^2}\\ &-\frac{\sqrt{2\lambda}\,e^{-ip}\sin^2(p/2)[4\,c\sin^2(p/2) + \sin(p/2) - 5\,i\cos(p/2)]}{e^2} + O\left(\lambda^0\right)\,.\end{aligned} \tag{5.8}$$

JHEP07(2009)034

Therefore, all these contributions together give in the end

$$\delta\epsilon^{\mu} = \left[-8\sqrt{2\lambda}\sin^3\left(\frac{p}{2}\right) - \frac{16\sin\left(\frac{p}{2}\right)}{\pi} + 8\,i\sin\left(\frac{p}{2}\right) - 16\,c\sin^3\left(\frac{p}{2}\right)\right.$$
$$\left. + \frac{4\sqrt{2}L}{\sqrt{\lambda}}\left(i\cos\left(\frac{p}{2}\right) - 2\,c\sin^2\left(\frac{p}{2}\right)\right)\right] e^{-2-\frac{L}{\sqrt{2\lambda}\sin(p/2)}} + O\left(\frac{1}{\sqrt{\lambda}}\right) . \tag{5.9}$$

If we compare this result with the μ-term for a giant magnon in $\mathcal{N}=4$ SYM [35]

$$\delta\epsilon^{\mu}_{SYM} = \left[-16\,g\sin^3\left(\frac{p}{2}\right) - \frac{16\sin\left(\frac{p}{2}\right)}{\pi} + 8\,i\sin\left(\frac{p}{2}\right) - 8\,i\sin(p)\right.$$
$$\left. + \frac{4\,i\,L}{g}\cos\left(\frac{p}{2}\right)\right] e^{-2-\frac{L}{2g\sin(p/2)}} + O\left(\frac{1}{g}\right) , \tag{5.10}$$

we notice that, differently to the classical contribution (section 2), the substitution $g \to h(\lambda)$ is no longer enough to match the two results. In fact the relevant difference is the term proportional to $\sin(p)$, that is missing in (5.9) because of the different nature of the S-matrix contribution. Furthermore it is an obvious fact that the terms in (5.9) which are proportional to c could be obtained by substituting $g/\sqrt{2}$ with $h(\lambda)$ the corresponding result of $\mathcal{N}=4$ SYM

$$\delta\epsilon^{\mu} \simeq -\frac{16}{e^2}\,h(\lambda)\sin^3\left(\frac{p}{2}\right) e^{-\frac{L}{2h(\lambda)\sin(p/2)}} \tag{5.11}$$

and then expanding $h(\lambda)$ as in (5.1).

For the $\mathbb{CP}^1$ giant magnon we follow the same steps of calculation again, hence we omit the details and give directly the result for the μ-term up to L/λ order:

$$\delta\epsilon^{\mu}_{s} = e^{-\frac{L}{\sqrt{2\lambda}\sin(p/2)}}\left[\frac{2\,i\sin(p/2)}{e} + \frac{1}{\sqrt{2\lambda}\,e\sin(p/2)}\left(\frac{2\,i}{\pi} - e^{i\frac{p}{2}} - 1\right) + \right.$$
$$\left. + \frac{L}{\lambda\,e}\left(2\,i\,c + \frac{\cos(p/2)}{\sin^2(p/2)}\right)\right] + O\left(\frac{1}{\lambda}\right) . \tag{5.12}$$

So, if we take the real part, for the considerations made in the previous section, we have at this order the following not vanishing terms

$$\mathrm{Re}\left[\delta\epsilon^{\mu}_{s}\right] = e^{-\frac{L}{\sqrt{2\lambda}\sin(p/2)}}\left[-\frac{\cos(p/2)+1}{\sqrt{2\lambda}\,e\sin(p/2)} + \frac{L\cos(p/2)}{\lambda\,e\sin^2(p/2)}\right] + O\left(\frac{1}{\lambda}\right) . \tag{5.13}$$

Of course, as far as the term proportional to c is concerned, it could be simply obtained from the expansion of the leading term, if we suppose that its dependence on the coupling constant is given by the strong coupling expansion of $h(\lambda)$:

$$\frac{2\,i\sin(p/2)}{e}\,e^{-\frac{L}{2h(\lambda)\sin(p/2)}} = \frac{2\,i\sin(p/2)}{e}\,e^{-\frac{L}{\sqrt{2\lambda}\sin(p/2)}}\left(1 + \frac{c\,L}{\lambda\sin(p/2)}\right) + O\left(\frac{L}{\lambda^{3/2}}\right) . \tag{5.14}$$

At this point, it would be of interest a comparison of the next-to-leading results (5.9) and (5.12) with possible algebraic curve results, derived along the lines of [35], in order to

JHEP07(2009)034

give a check and a constraint on the value of c. Yet, by now we do have only heuristic preliminary results which are though promising. Therefore, the calculation of these quantum corrections by this completely different technique is currently under investigation and may be the subject of a future publication.

6 Conclusions

In this paper we compute the classical and the first quantum finite-size corrections to the energy of giant magnons in the $SU(2) \times SU(2)$ sector of $\mathcal{N} = 6$ superconformal Chern-Simons theory. Therefore we provide a check of the string result [23], of the algebraic curve result [26] and then a test for the all-loop Bethe Ansatz [10], for the S-matrix [11] and, more generally, for the AdS_4/CFT_3 correspondence [5].

We have proposed some generalised Lüscher formulae heuristically derived from [32, 33, 38, 39]. We applied them in the one-particle case in order to calculate finite size correction to the energy of the so-called "small" giant magnon. It turns out a perfect agreement with algebraic curve calculations [26] for the F-term, while we propose a prediction for the μ-term that needs some deeper understanding, as we explain in the main text. For the giant magnon that lives on $\mathbb{R} \times S^2 \times S^2$, we applied the formulae for the case of multi-particle states considering the strong coupling limit, where the interactions between elementary magnons is dominant and one can neglect the contributions coming from all the bound states of the theory. In particular, our μ-term is in perfect agreement with the string result by [23] and the F-term matches the algebraic curve result by [26] for the "big" giant magnon.

Indeed it would be extremely interesting to investigate for example the bound states S-matrix and the mirror [52] counterpart of the sector we considered, in order to study wrapping effects also at weak coupling (see [39] for $\mathcal{N} = 4$ SYM) and finite-size effects for dyonic giant magnons in $\mathbb{CP}^3$ (see [27, 50] for string computations and [51] for string and Lüscher-terms results in $\mathcal{N} = 4$ SYM).

On the other hand, further investigations about sub-leading finite-size corrections at strong coupling could be - as mentioned above - an interesting future research direction as well.

Acknowledgments

We are especially indebted to G. Grignani and R. Suzuki and we also thank F. Ravanini, M. Rossi, I. Shenderovich, R. Tateo for insightful discussions and correspondence. We acknowledge the INFN grant "Iniziativa specifica PI14" *Topics in non-perturbative gauge dynamics in field and string theory* for travel financial support, and the University PRIN 2007JHLPEZ "Fisica Statistica dei Sistemi Fortemente Correlati all'Equilibrio e Fuori Equilibrio: Risultati Esatti e Metodi di Teoria dei Campi". We thank the Galileo Galilei Institute for Theoretical Physics for the hospitality and the INFN for partial support during the completion of this work.

JHEP07(2009)034

A Reconsidering the algebraic curve F-term for the "big" and "small" GM

The equation (5.5) in [26] reads

$$\delta\epsilon_{1-loop} = -\sum_{ij}\gamma_{ij}(-1)^{F_{ij}}\oint_{\mathbb{U}}\frac{dx}{4i}\frac{p_i'-p_j'}{2\pi}\cot\left(\frac{p_i-p_j}{2}\right)\Omega(x) \tag{A.1}$$

Let us consider this expression when L, namely Δ, is large; then, since the quasimomenta have the following expression in terms of Δ:

$$p_i \simeq \frac{\Delta\, x}{x^2-1}\,, \tag{A.2}$$

also the quasimomenta are large in this limit. Therefore we take the expansion of the cotangent when the p_i are large and we ought to distinguish the two cases$x \in \mathbb{U}^{\pm}$:

$$\cot\left(\frac{p_i-p_j}{2}\right) = \pm i\left(1+2\,e^{\mp i(p_i-p_j)}+\ldots\right), \tag{A.3}$$

where all the equilevel simbols are considered in the same expression. Only the exponential part of (A.3) contributes in the integral (A.1), then, after an integration by parts it becomes

$$\delta\epsilon_{1-loop} = -\oint_{\mathbb{U}^+}\frac{dx}{4i\pi}\,\partial_x\Omega(x)\sum_{ij}\gamma_{ij}(-1)^{F_{ij}}e^{-i(p_i-p_j)} - \oint_{\mathbb{U}^-}\frac{dx}{4i\pi}\,\partial_x\Omega(x)\sum_{ij}\gamma_{ij}(-1)^{F_{ij}}e^{i(p_i-p_j)}, \tag{A.4}$$

Now, one can easily verify, once made explicit the quasimomenta in terms of x and $x^{\pm}$, that the two integrals above give the same contribution, in such way that one can perform the saddle-point evaluation on the same integral we obtain from the Lüscher term calculations in the main text:

$$\delta\epsilon_{1-loop} = -\oint_{\mathbb{U}^+}\frac{dx}{2i\pi}\,\partial_x\Omega(x)\sum_{ij}\gamma_{ij}(-1)^{F_{ij}}e^{-i(p_i-p_j)} \tag{A.5}$$

where

$$\sum_{ij}\gamma_{ij}(-1)^{F_{ij}}e^{-i(p_i-p_j)} = 4\,e^{-i\sqrt{2}\frac{(\Delta-L)}{\sqrt{\lambda}}}\left[\left(\frac{x-x_p^-}{x\,x_p^- - 1}\right)^2 - 1\right] \tag{A.6}$$

in the case of the "big" GM, and

$$\sum_{ij}\gamma_{ij}(-1)^{F_{ij}}e^{-i(p_i-p_j)} = 4\,e^{-i\sqrt{2}\frac{(\Delta_s-L)}{\sqrt{\lambda}}}\left(\frac{x-x_p^-}{x\,x_p^- - 1} - 1\right) \tag{A.7}$$

in the case of the "small" GM. These expressions match exactly our S-matrix contributions in (3.7) and (4.7).

JHEP07(2009)034

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113] [hep-th/9711200] [SPIRES];
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [SPIRES];
E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [SPIRES].

[2] L.N. Lipatov, *High-energy asymptotics of multicolor QCD and exactly solvable lattice models*, hep-th/9311037 [SPIRES].

[3] J.A. Minahan and K. Zarembo, *The Bethe-ansatz for $N = 4$ super Yang-Mills*, *JHEP* **03** (2003) 013 [hep-th/0212208] [SPIRES];
N. Beisert and M. Staudacher, *Long-range $PSU(2,2|4)$ Bethe ansaetze for gauge theory and strings*, *Nucl. Phys.* **B 727** (2005) 1 [hep-th/0504190] [SPIRES].

[4] I. Bena, J. Polchinski and R. Roiban, *Hidden symmetries of the $AdS_5 \times S^5$ superstring*, *Phys. Rev.* **D 69** (2004) 046002 [hep-th/0305116] [SPIRES].

[5] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [SPIRES].

[6] J.A. Minahan and K. Zarembo, *The Bethe ansatz for superconformal Chern-Simons*, *JHEP* **09** (2008) 040 [arXiv:0806.3951] [SPIRES].

[7] D. Bak and S.-J. Rey, *Integrable spin chain in superconformal Chern-Simons theory*, *JHEP* **10** (2008) 053 [arXiv:0807.2063] [SPIRES].

[8] G. Arutyunov and S. Frolov, *Superstrings on $AdS_4 \times CP^3$ as a Coset σ-model*, *JHEP* **09** (2008) 129 [arXiv:0806.4940] [SPIRES].

[9] B. Stefanski Jr., *Green-Schwarz action for Type IIA strings on $AdS_4 \times CP^3$*, *Nucl. Phys.* **B 808** (2009) 80 [arXiv:0806.4948] [SPIRES].

[10] N. Gromov and P. Vieira, *The all loop AdS_4/CFT_3 Bethe ansatz*, *JHEP* **01** (2009) 016 [arXiv:0807.0777] [SPIRES].

[11] C. Ahn and R.I. Nepomechie, *N=6 super Chern-Simons theory S-matrix and all-loop Bethe ansatz equations*, *JHEP* **09** (2008) 010 [arXiv:0807.1924] [SPIRES].

[12] M.J. Martins and C.S. Melo, *The Bethe ansatz approach for factorizable centrally extended S-matrices*, *Nucl. Phys.* **B 785** (2007) 246 [hep-th/0703086] [SPIRES].

[13] T. McLoughlin and R. Roiban, *Spinning strings at one-loop in $AdS_4 \times P^3$*, *JHEP* **12** (2008) 101 [arXiv:0807.3965] [SPIRES].

[14] L.F. Alday, G. Arutyunov and D. Bykov, *Semiclassical quantization of spinning strings in $AdS_4 \times CP^3$*, *JHEP* **11** (2008) 089 [arXiv:0807.4400] [SPIRES].

[15] C. Krishnan, *AdS_4/CFT_3 at one loop*, *JHEP* **09** (2008) 092 [arXiv:0807.4561] [SPIRES].

[16] N. Gromov and V. Mikhaylov, *Comment on the scaling function in $AdS^4 \times CP_3$*, *JHEP* **04** (2009) 083 [arXiv:0807.4897] [SPIRES].

[17] T. McLoughlin, R. Roiban and A.A. Tseytlin, *Quantum spinning strings in $AdS_4 \times CP^3$: testing the Bethe Ansatz proposal*, *JHEP* **11** (2008) 069 [arXiv:0809.4038] [SPIRES].

JHEP07(2009)034

[18] D.M. Hofman and J.M. Maldacena, *Giant magnons*, *J. Phys.* **A 39** (2006) 13095 [hep-th/0604135] [SPIRES].

[19] T. Nishioka and T. Takayanagi, *On Type IIA Penrose limit and $N = 6$ Chern-Simons theories*, *JHEP* **08** (2008) 001 [arXiv:0806.3391] [SPIRES].

[20] D. Gaiotto, S. Giombi and X. Yin, *Spin chains in $N = 6$ superconformal Chern-Simons-Matter theory*, *JHEP* **04** (2009) 066 [arXiv:0806.4589] [SPIRES].

[21] G. Grignani, T. Harmark and M. Orselli, *The* $\mathrm{SU}(2) \times \mathrm{SU}(2)$ *sector in the string dual of $N = 6$ superconformal Chern-Simons theory*, *Nucl. Phys.* **B 810** (2009) 115 [arXiv:0806.4959] [SPIRES].

[22] D. Berenstein and D. Trancanelli, *Three-dimensional $N = 6$ SCFT's and their membrane dynamics*, *Phys. Rev.* **D 78** (2008) 106009 [arXiv:0808.2503] [SPIRES].

[23] G. Grignani, T. Harmark, M. Orselli and G.W. Semenoff, *Finite size giant magnons in the string dual of $N = 6$ superconformal Chern-Simons theory*, *JHEP* **12** (2008) 008 [arXiv:0807.0205] [SPIRES].

[24] D. Astolfi, V.G.M. Puletti, G. Grignani, T. Harmark and M. Orselli, *Finite-size corrections in the* $\mathrm{SU}(2) \times \mathrm{SU}(2)$ *sector of type IIA string theory on $AdS_4 \times CP^3$*, *Nucl. Phys.* **B 810** (2009) 150 [arXiv:0807.1527] [SPIRES].

[25] C. Ahn and P. Bozhilov, *Finite-size effects of membranes on $AdS_4 \times S_7$*, *JHEP* **08** (2008) 054 [arXiv:0807.0566] [SPIRES].

[26] I. Shenderovich, *Giant magnons in AdS_4/CFT_3: dispersion, quantization and finite–size corrections*, arXiv:0807.2861 [SPIRES].

[27] C. Ahn, P. Bozhilov and R.C. Rashkov, *Neumann-Rosochatius integrable system for strings on $AdS_4 \times CP^3$*, *JHEP* **09** (2008) 017 [arXiv:0807.3134] [SPIRES].

[28] B.-H. Lee, K.L. Panigrahi and C. Park, *Spiky strings on $AdS_4 \times \mathbf{CP}^3$*, *JHEP* **11** (2008) 066 [arXiv:0807.2559] [SPIRES].

[29] M. Lüscher, *On a relation between finite size effects and elastic scattering processes*, lecture given at *Cargese Summer Inst.*, Cargese France September 1–15 1983; *Volume dependence of the energy spectrum in massive quantum field theories. 1. Stable particle states*, *Commun. Math. Phys.* **104** (1986) 177 [SPIRES];
T.R. Klassen and E. Melzer, *On the relation between scattering amplitudes and finite size mass corrections in QFT*, *Nucl. Phys.* **B 362** (1991) 329 [SPIRES].

[30] J. Ambjørn, R.A. Janik and C. Kristjansen, *Wrapping interactions and a new source of corrections to the spin-chain/string duality*, *Nucl. Phys.* **B 736** (2006) 288 [hep-th/0510171] [SPIRES].

[31] A. Rej, D. Serban and M. Staudacher, *Planar $N = 4$ gauge theory and the Hubbard model*, *JHEP* **03** (2006) 018 [hep-th/0512077] [SPIRES];
G. Feverati, D. Fioravanti, P. Grinza and M. Rossi, *Hubbard's adventures in $N = 4$ SYM-land? Some non- perturbative considerations on finite length operators*, *J. Stat. Mech.* (2007) P02001 [hep-th/0611186] [SPIRES].

[32] R.A. Janik and T. Lukowski, *Wrapping interactions at strong coupling – the giant magnon*, *Phys. Rev.* **D 76** (2007) 126008 [arXiv:0708.2208] [SPIRES].

[33] N. Gromov, S. Schäfer-Nameki and P. Vieira, *Quantum wrapped giant magnon*, *Phys. Rev.* **D 78** (2008) 026006 [arXiv:0801.3671] [SPIRES].

JHEP07(2009)034

[34] M.P. Heller, R.A. Janik and T. Lukowski, *A new derivation of Lüscher F-term and fluctuations around the giant magnon*, *JHEP* **06** (2008) 036 [arXiv:0801.4463] [SPIRES].

[35] N. Gromov, S. Schäfer-Nameki and P. Vieira, *Efficient precision quantization in AdS/CFT*, *JHEP* **12** (2008) 013 [arXiv:0807.4752] [SPIRES].

[36] J.A. Minahan and O. Ohlsson Sax, *Finite size effects for giant magnons on physical strings*, *Nucl. Phys.* **B 801** (2008) 97 [arXiv:0801.2064] [SPIRES].

[37] J. Penedones and P. Vieira, *Toy models for wrapping effects*, *JHEP* **08** (2008) 020 [arXiv:0806.1047] [SPIRES].

[38] Y. Hatsuda and R. Suzuki, *Finite-size effects for multi-magnon states*, *JHEP* **09** (2008) 025 [arXiv:0807.0643] [SPIRES].

[39] Z. Bajnok and R.A. Janik, *Four-loop perturbative Konishi from strings and finite size effects for multiparticle states*, *Nucl. Phys.* **B 807** (2009) 625 [arXiv:0807.0399] [SPIRES].

[40] N. Beisert, *The* SU(2|2) *dynamic S-matrix*, *Adv. Theor. Math. Phys.* **12** (2008) 945 [hep-th/0511082] [SPIRES].

[41] N. Beisert, *The analytic Bethe Ansatz for a chain with centrally extended su*(2|2) *symmetry*, *J. Stat. Mech.* (2007) P01017 [nlin/0610017] [SPIRES].

[42] G. Arutyunov, S. Frolov and M. Staudacher, *Bethe ansatz for quantum strings*, *JHEP* **10** (2004) 016 [hep-th/0406256] [SPIRES].

[43] N. Beisert, B. Eden and M. Staudacher, *Transcendentality and crossing*, *J. Stat. Mech.* (2007) P01021 [hep-th/0610251] [SPIRES].

[44] R. Hernandez and E. Lopez, *Quantum corrections to the string Bethe ansatz*, *JHEP* **07** (2006) 004 [hep-th/0603204] [SPIRES].

[45] N. Beisert, R. Hernandez and E. Lopez, *A crossing-symmetric phase for* $AdS_5 \times S^5$ *strings*, *JHEP* **11** (2006) 070 [hep-th/0609044] [SPIRES].

[46] N. Dorey, D.M. Hofman and J.M. Maldacena, *On the singularities of the magnon S-matrix*, *Phys. Rev.* **D 76** (2007) 025011 [hep-th/0703104] [SPIRES].

[47] G. Arutyunov, S. Frolov and M. Zamaklar, *Finite-size effects from giant magnons*, *Nucl. Phys.* **B 778** (2007) 1 [hep-th/0606126] [SPIRES].

[48] D. Astolfi, V. Forini, G. Grignani and G.W. Semenoff, *Gauge invariant finite size spectrum of the giant magnon*, *Phys. Lett.* **B 651** (2007) 329 [hep-th/0702043] [SPIRES].

[49] G. Arutyunov, S. Frolov and M. Zamaklar, *The Zamolodchikov-Faddeev algebra for* $AdS_5 \times S^5$ *superstring*, *JHEP* **04** (2007) 002 [hep-th/0612229] [SPIRES].

[50] S. Ryang, *Giant magnon and spike solutions with two spins in* $AdS_4 \times CP^3$, *JHEP* **11** (2008) 084 [arXiv:0809.5106] [SPIRES].

[51] Y. Hatsuda and R. Suzuki, *Finite-size effects for Dyonic giant magnons*, *Nucl. Phys.* **B 800** (2008) 349 [arXiv:0801.0747] [SPIRES].

[52] G. Arutyunov and S. Frolov, *On string S-matrix, bound states and TBA*, *JHEP* **12** (2007) 024 [arXiv:0710.1568] [SPIRES]; *The S-matrix of string bound states*, *Nucl. Phys.* **B 804** (2008) 90 [arXiv:0803.4323] [SPIRES].

[53] Y. Koma and M. Koma, *On the finite size mass shift formula for stable particles*, *Nucl. Phys.* **B 713** (2005) 575 [hep-lat/0406034] [SPIRES]; *Finite size mass shift formula for stable particles revisited*, *Nucl. Phys. Proc. Suppl.* **140** (2005) 329 [hep-lat/0409002] [SPIRES]; *More on the finite size mass shift formula for stable particles*, hep-lat/0504009 [SPIRES].

JHEP07(2009)034